Payment Uniqueness

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Abstract

An agent with a private signal receives a separable payoff $\int v_k(s)\, q_k\, d\nu(k) - p$ in an incentive-compatible mechanism. We prove that the payment is unique if and only if the infimum of the round-trip cost vanishes. Loosely, valuations and allocations cannot jump together. On an interval type space the condition reduces to no common discontinuities and the payment takes the Riemann–Stieltjes form $p(s) = p(\underline{s}) + \sum_k \int v_k\, dq_k$. No differentiability is needed. Classical revenue equivalence is the special case of $v(s) = s$. We apply the theorem to the one-insider common-value model of Wilson [1967] and Engelbrecht-Wiggans et al. [1983]. In every bid-distribution-symmetric equilibrium, the insider's winning probability is format-invariant. Payment uniqueness then pins down revenue. The proof decomposes revenue equivalence into payment uniqueness and allocation invariance, separating where equivalence holds from where it fails.

Keywords: Payment uniqueness, revenue equivalence, type graphs, separable payoffs, mechanism design.

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1 Introduction

An agent privately observes a signal s drawn from an ordered type space $\Sigma \subset \mathbb{R}$ and participates in a mechanism that allocates goods indexed by a σ-finite measure space $(\mathcal{K}, \nu)$. The agent's payoff takes the separable form $\int_{\mathcal{K}} v_k(s)\, q_k\, d\nu(k) - p$, where $v_k(s)$ is a valuation, q_k is an allocation probability, and p is a scalar payment. If each pair (v_k, q_k) is co-monotone (both increasing or both decreasing in s), the payment is uniquely determined up to a constant if and only if the infimum of the round-trip cost over finite chains vanishes between every pair of types. Loosely, the valuation and allocation cannot jump together. When Σ is an interval and there are finitely many goods, the condition reduces to v_k and q_k having no common discontinuities for each k, and the payment takes the Riemann–Stieltjes form

$$p(s) = p(\underline{s}) + \sum_{k=1}^{K} \int_{\underline{s}}^{s} v_k\, dq_k. \tag{1}$$

The integral formula is the unique incentive-compatible payment when it is well-defined. When Σ is a finite set of types, the condition becomes a testable step-by-step requirement.

The result depends only on the separable payoff structure, not on what the v_k and q_k represent. The mechanism can be an auction, a multi-unit sale, a nonlinear pricing scheme, or a hiring rule, among many others. Each valuation can be a private value, a conditional expectation of a common value, or any monotone function of the signal. Classical revenue equivalence, the payment identity of Vickrey [1961], Myerson [1981], and Riley and Samuelson [1981], is the special case of one good with $v(s) = s$. Indeed, our result shows that what is usually presented as a theorem about equilibrium revenue is, at root, a theorem about separable payoffs.

The proof builds on the shortest-path framework of Heydenreich et al. [2009, hereafter HMUV]. They show that the node potential on any directed graph is unique if and only if shortest-path distances are anti-symmetric. This is an abstract criterion, but it is often not transparent where anti-symmetry holds. Specifically, verifying it requires computing shortest-path distances in a graph with uncountably many nodes. We show that for separable payoffs, anti-symmetry reduces to a single condition on the primitives (v_k, q_k), namely the vanishing of the round-trip cost. We demonstrate that the round-trip cost equals the anti-symmetry gap. This yields both necessity and sufficiency. Necessity (no other payment exists) is new relative to the envelope-theorem approach of Milgrom and Segal [2002], and the if-and-only-if identifies separability, not smoothness or absolute continuity, as the condition for payment uniqueness. The result complements Archer and Tardos [2001], who show that an allocation is implementable (admits at least one incentive-compatible payment) if and only if it is monotone. We show that the payment is unique (admits at most one) if and only if the infimum of the round-trip cost vanishes. Together, these results fully characterize the allocation-payment relationship in single-parameter environments.

Payment uniqueness applies, for instance, to the one-insider common-value model of Wilson [1967, 1969] and Engelbrecht-Wiggans et al. [1983, hereafter EMW], in which a single informed bidder competes against uninformed bidders. EMW characterized the equilibrium of the first-price auction, but revenue equivalence across formats was not established. Payment uniqueness resolves the problem. In every bid-distribution-symmetric (BDS) equilibrium, the insider's winning probability does not depend on the auction format (allocation invariance). Payment uniqueness then pins down the expected payment, and revenue in every standard auction.[1] This result is

[1]This model has strong empirical grounding in the sale of drainage tracts on the U.S. outer continental shelf. Hendricks and Porter [1988], Hendricks et al. [1994], and Porter [1995] confirmed EMW's first-price predictions with field data. Hendricks and Porter [2014] survey these findings.

new, but it is also one application of the general theorem. The same argument yields payoff equivalence in any common-value mechanism, and the decomposition into payment uniqueness and allocation invariance applies to private-value, common-value, and interdependent-value settings alike. With multiple insiders, payment uniqueness still holds for each bidder, but allocation invariance generically fails. An exact formula (Proposition 15) quantifies the revenue difference across formats.

2 Setup and Main Result

An agent observes a private signal s drawn from an ordered type space $\Sigma \subset \mathbb{R}$ with least element $\underline{s}$ and greatest element $\bar{s}$. The mechanism allocates goods indexed by a σ-finite measure space $(\mathcal{K}, \nu)$. For each good $k \in \mathcal{K}$, let $v_k(s)$ denote the agent's valuation and $q_k(s) \in [0,1]$ the allocation probability. The agent's expected surplus is

$$\pi(s) = \int_{\mathcal{K}} v_k(s)\, q_k(s)\, d\nu(k) \ - \ p(s),$$

where $p(s)$ is the interim expected payment. We assume that $k \mapsto v_k(s)q_k(s)$ is ν-integrable for each $s \in \Sigma$. When $\mathcal{K} = \{1, \ldots, K\}$ with counting measure, the integral reduces to $\sum_{k=1}^{K} v_k(s)q_k(s)$.

Incentive compatibility requires that reporting truthfully is optimal. For all $s, t \in \Sigma$,

$$\int_{\mathcal{K}} v_k(s)\, q_k(s)\, d\nu(k) - p(s) \ \geq \ \int_{\mathcal{K}} v_k(s)\, q_k(t)\, d\nu(k) - p(t).$$

Writing the same inequality for type t and rearranging,

$$p(t) - p(s) \ \leq \ \int_{\mathcal{K}} v_k(t)\, [q_k(t) - q_k(s)]\, d\nu(k) \ =: \ \ell_{st}. \tag{2}$$

Grosskopf et al. [2018] provide laboratory evidence for the first-price format and Grosskopf et al. [2010] do the same for all-pay auctions.

The payoff is *separable* such that the signal enters only through the valuations $v_k(s)$, and linearly.

The incentive compatibility constraints (2) have a graph-theoretic interpretation. Define the *type graph* $\mathcal{T}$ with node set Σ and arc lengths ℓ_{st}. A *node potential* is a function $p\colon \Sigma \to \mathbb{R}$ satisfying $p(t) - p(s) \leq \ell_{st}$ for all s, t. Incentive compatibility says that the payment p is a node potential. The question is whether p is the *only* node potential (up to a constant), or whether other incentive-compatible payments exist for the same allocation.

For each good $k \in \mathcal{K}$, call the pair (v_k, q_k) *co-monotone* on Σ if both v_k and q_k are increasing on Σ, or both are decreasing on Σ. For $s < t$ in Σ, the *round-trip cost* along any finite chain $s = \sigma_0 < \sigma_1 < \cdots < \sigma_m = t$ in Σ is

$$\mathcal{C}(\sigma_0, \ldots, \sigma_m) \;:=\; \sum_{i=1}^{m} \int_{\mathcal{K}} [v_k(\sigma_i) - v_k(\sigma_{i-1})][q_k(\sigma_i) - q_k(\sigma_{i-1})]\, d\nu(k). \tag{3}$$

Under co-monotonicity, each factor $[v_k(\sigma_i) - v_k(\sigma_{i-1})][q_k(\sigma_i) - q_k(\sigma_{i-1})]$ is non-negative, so $\mathcal{C} \geq 0$ for every chain.

Theorem 1 (Payment Uniqueness)**.** *Let (v_k, q_k) be co-monotone on Σ for ν-a.e. $k \in \mathcal{K}$. In any incentive-compatible mechanism satisfying* (2)*, the payment is pinned down uniquely if and only if, for every $s < t$ in Σ, the infimum of the round-trip cost over all finite chains vanishes*

$$\inf\left\{\mathcal{C}(\sigma_0, \ldots, \sigma_m) : m \geq 1,\ s \leq \sigma_0 < \cdots < \sigma_m \leq t,\ \sigma_i \in \Sigma\right\} = 0. \tag{4}$$

When uniqueness holds, $p(s) = p(\underline{s}) + \operatorname{dist}(\underline{s}, s)$, where

$$\operatorname{dist}(a, b) := \inf\left\{\sum_{i=1}^{n} \ell_{z_{i-1}, z_i} : n \geq 1,\ a = z_0, z_1, \ldots, z_n = b,\ z_i \in \Sigma\right\}$$

is the shortest-path distance in the type graph. This holds regardless of the mechanism, the number of other agents, and the information and strategies of opponents.

The proof has three steps: monotone path optimality (Lemma 2), the round-trip identity (Lemma 3), and the HMUV uniqueness criterion.

Lemma 2. *Under the conditions of Theorem 1, for $a < b < c$ in Σ*

(a) $\ell_{ab} + \ell_{bc} \le \ell_{ac}$. *(Forward subdivision shortens.)*

(b) $\ell_{ac} + \ell_{cb} \ge \ell_{ab}$. *(Forward-backward detour lengthens.)*

(c) $\ell_{cb} + \ell_{ba} \le \ell_{ca}$. *(Backward subdivision shortens.)*

(d) $\ell_{ca} + \ell_{ab} \ge \ell_{cb}$. *(Backward-forward detour lengthens.)*

Consequently, monotone paths attain the infimum in $\mathrm{dist}(s, t)$.

Proof. For $a < b < c$, co-monotonicity gives $[v_k(b) - v_k(c)][q_k(b) - q_k(a)] \le 0$ pointwise in k. Whether both functions increase or both decrease, the two factors have opposite signs. Integrating against ν

(a) $\ell_{ab} + \ell_{bc} - \ell_{ac} = \int_{\mathcal{K}} [v_k(b) - v_k(c)][q_k(b) - q_k(a)]\, d\nu(k) \le 0$.

(b) $\ell_{ac} + \ell_{cb} - \ell_{ab} = \int_{\mathcal{K}} [v_k(c) - v_k(b)][q_k(c) - q_k(a)]\, d\nu(k) \ge 0$.

(c) $\ell_{cb} + \ell_{ba} - \ell_{ca} = \int_{\mathcal{K}} [v_k(b) - v_k(a)][q_k(b) - q_k(c)]\, d\nu(k) \le 0$.

(d) $\ell_{ca} + \ell_{ab} - \ell_{cb} = \int_{\mathcal{K}} [v_k(b) - v_k(a)][q_k(c) - q_k(a)]\, d\nu(k) \ge 0$.

Any non-monotone path contains a consecutive triple where the middle node reverses direction. At a peak ($r_1 < r_2 > r_3$), inequality (b) gives $\ell_{r_1,r_2} + \ell_{r_2,r_3} \ge \ell_{r_1,r_3}$. At a valley ($r_1 > r_2 < r_3$), inequality (d) gives the same. Removing the middle node does not increase total length. Iterating removes all reversals. □

Lemma 3 (Round-Trip Identity)**.** *Under the conditions of Theorem 1, for $s < t$ in Σ*

$$\operatorname{dist}(s,t) + \operatorname{dist}(t,s) \;=\; \inf\bigl\{\mathcal{C}(\sigma_0,\dots,\sigma_m) : m \geq 1,\ s \leq \sigma_0 < \cdots < \sigma_m \leq t,\ \sigma_i \in \Sigma\bigr\}.$$

Proof. By Lemma 2, $\operatorname{dist}(s,t)$ is the infimum over increasing chains of the forward path length $L^+(P) := \sum_i \ell_{\sigma_{i-1},\sigma_i}$, and $\operatorname{dist}(t,s)$ is the infimum over decreasing chains of the backward path length $L^-(P) := \sum_i \ell_{\sigma_i,\sigma_{i-1}}$. A direct computation gives $L^+(P) + L^-(P) = \mathcal{C}(P)$ for any partition P.

Lower bound. For any P, $L^+(P) \geq \operatorname{dist}(s,t)$ and $L^-(P) \geq \operatorname{dist}(t,s)$, so $\mathcal{C}(P) \geq \operatorname{dist}(s,t) + \operatorname{dist}(t,s)$. Taking the infimum, $\inf_P \mathcal{C}(P) \geq \operatorname{dist}(s,t) + \operatorname{dist}(t,s)$.

Upper bound. For any $\varepsilon > 0$, choose an increasing chain P_1 with $L^+(P_1) < \operatorname{dist}(s,t) + \varepsilon/2$ and a decreasing chain P_2 with $L^-(P_2) < \operatorname{dist}(t,s) + \varepsilon/2$. Let $P = P_1 \cup P_2$. By Lemma 2(a), $L^+(P) \leq L^+(P_1)$ (refinement shortens forward paths). By (c), $L^-(P) \leq L^-(P_2)$. Hence $\mathcal{C}(P) = L^+(P) + L^-(P) < \operatorname{dist}(s,t) + \operatorname{dist}(t,s) + \varepsilon$. □

Proof of Theorem 1. By Lemma 3, condition (4) is equivalent to $\operatorname{dist}(s,t) + \operatorname{dist}(t,s) = 0$ for all s,t in Σ. Heydenreich et al. [2009, Theorem 1] prove that the node potential on any directed graph is unique up to a constant if and only if this anti-symmetry condition holds. The payment formula $p(s) = p(\underline{s}) + \operatorname{dist}(\underline{s}, s)$ follows from the same theorem. □

The round-trip cost measures the gap between forward and backward shortest-path distances. More precisely, $\inf \mathcal{C} = \operatorname{dist}(s,t) + \operatorname{dist}(t,s)$ (Lemma 3), so condition (4) is the anti-symmetry condition $\operatorname{dist}(s,t) + \operatorname{dist}(t,s) = 0$ that characterizes uniqueness. The infimum cannot be dropped because on a coarse chain, $\mathcal{C}$ may be strictly positive even when uniqueness holds. (For instance, $v(s) = q(s) = s$ on

$[0,1]$ gives $\mathcal{C}(0,1) = 1$, but $\mathcal{C} \to 0$ under refinement and the payment $p(s) = s^2/2$ is unique.) On a finite type space, no refinement is possible and the infimum reduces to $\mathcal{C}$ on the finest chain, giving the step-by-step condition of Corollary 5.

Co-monotonicity is a consequence of incentive compatibility when $\mathcal{K}$ is a singleton. Two-sided IC requires $[v_k(t) - v_k(s)][q_k(t) - q_k(s)] \geq 0$ for all s, t, forcing q_k to be co-monotone with v_k whenever v_k is strictly monotone.[2] The theorem does not restrict the interpretations of v_k and q_k. Each valuation can be a private value, a conditional expectation, or any monotone function. Each allocation can be a winning probability, a quantity, or a hiring probability. The signal distribution plays no role.

When $\mathcal{K}$ is a singleton and v and q are differentiable, the envelope theorem gives $\pi'(s) = v'(s)q(s)$, from which p can be recovered. Milgrom and Segal [2002] extend this to require only absolute continuity, but their approach provides sufficiency, not necessity. When v is *strictly* increasing, a change of variables $\theta = v(s)$ reduces the problem to the private-value form $\theta\tilde{q} - \tilde{p}$, and the Milgrom–Segal theorem applies directly. This alternative derivation is valid for the one-insider application (Section 3), where we assume strict monotonicity.

The HMUV framework offers three advantages over the envelope-theorem approach. It provides necessity (no other incentive-compatible payment exists for the same allocation), it handles the general case (v_k merely monotone), and it accommodates allocations with jumps without regularity conditions.[3]

[2] With multiple goods, IC requires $\int_{\mathcal{K}} [v_k(t) - v_k(s)][q_k(t) - q_k(s)]\, d\nu(k) \geq 0$ but does not force each integrand to be non-negative pointwise. Co-monotonicity of each pair is therefore a restriction, though it is satisfied in all applications we consider.

[3] In procurement, the IC constraint is $p(t) - p(s) \leq v(s)[q(t) - q(s)]$ (note $v(s)$, not $v(t)$), giving arc lengths $\ell^P_{st} = v(s)[q(t) - q(s)]$. A parallel argument establishes anti-symmetry when v and q are co-monotone. For $a < b < c$, $\ell^P_{ab} + \ell^P_{bc} - \ell^P_{ac} = [v(b) - v(a)][q(c) - q(b)] \geq 0$.

2.1 Interval and Finite Type Spaces

While Theorem 1 is quite general, many applications involve interval type spaces with finitely many goods, where the round-trip condition and the shortest-path distances take familiar forms.

Corollary 4 (Interval Types)**.** *Let $\Sigma = [\underline{s}, \bar{s}]$ and $\mathcal{K} = \{1, \ldots, K\}$ with counting measure. The payment is uniquely determined (up to a constant) if and only if, for each k, v_k and q_k have no common discontinuities. When uniqueness holds,*

$$p(s) = p(\underline{s}) + \sum_{k=1}^{K} \int_{\underline{s}}^{s} v_k \, dq_k, \tag{5}$$

where $\int v_k \, dq_k$ is the Riemann–Stieltjes integral.

Proof. The round-trip cost on a partition $\sigma_0 < \cdots < \sigma_m$ is $\sum_i \sum_k [v_k(\sigma_i) - v_k(\sigma_{i-1})][q_k(\sigma_i) - q_k(\sigma_{i-1})]$. Each k-th component is the gap between the upper and lower Darboux–Stieltjes sums for $\int v_k \, dq_k$. For co-monotone (v_k, q_k), the Riemann–Stieltjes integral exists if and only if v_k and q_k have no common discontinuities (the sufficient condition is from [Apostol, 1974]). The Darboux gap converges to zero under refinement if and only if the integral exists, so the infimum of the round-trip cost over all partitions of $[\underline{s}, \bar{s}]$ is zero if and only if each integral exists. When it does, $\mathrm{dist}(\underline{s}, s) = \sum_k \int_{\underline{s}}^{s} v_k \, dq_k$. For each k, the forward path length along a partition is a right-endpoint Riemann–Stieltjes sum, which is an upper sum under co-monotonicity and converges to $\int v_k \, dq_k$ as the mesh refines. $\square$

The integral formula is the unique incentive-compatible payment when it is well-defined. When some v_k has a jump where q_k also jumps, the corresponding integral does not exist and the payment is not unique. For instance, with $K = 1$, let $v(s) = \mathbf{1}(s \geq 1/2)$ and $q(s) = \mathbf{1}(s \geq 1/2)$ on $[0, 1]$. Both jump at $1/2$, the integral $\int v \, dq$

is undefined, and incentive compatibility requires only $p(1/2) - p(0) \in [0,1]$. The condition of no common discontinuities holds whenever each v_k is continuous or each q_k is continuous In standard mechanism design applications, at least one holds.

Consider instead a two-unit auction where a firm of type $s \in [0,1]$ has marginal value $v_1(s) = s$ for the first unit and $v_2(s) = \mathbf{1}(s \geq 1/2)$ for the second (the firm needs a second unit only above a capacity threshold). If the efficient allocation awards the second unit to types above the threshold, so that $q_2(s) = \mathbf{1}(s \geq 1/2)$, then v_2 and q_2 have a common discontinuity at $s = 1/2$, and Corollary 4 says the payment is not unique. The width of the IC interval at $s = 1/2$ is $[v_2(1/2) - v_2(1/2^-)][q_2(1/2) - q_2(1/2^-)] = 1$. Different auction formats can implement the same allocation rule but charge different amounts for the second unit. The first unit contributes no non-uniqueness (v_1 is continuous), consistent with the good-by-good nature of the condition.

Corollary 5 (Finite Types)**.** *Let $\Sigma = \{s_1 < \cdots < s_n\}$ be a finite set of types. The payment is unique (up to a constant) if and only if, for each k and each consecutive pair (s_{i-1}, s_i), at most one of v_k and q_k changes*

$$[v_k(s_i) - v_k(s_{i-1})][q_k(s_i) - q_k(s_{i-1})] = 0 \quad \textit{for all } i = 1, \ldots, n-1 \textit{ and for } \nu \textit{ almost every } k.$$

When uniqueness holds, $p(s_i) = p(s_1) + \sum_{j=1}^{i-1} \ell_{s_j, s_{j+1}}$.

Proof. On a finite set, the finest chain between any $s_j < s_\ell$ uses all intermediate points $s_j, s_{j+1}, \ldots, s_\ell$. Since no further refinement is possible, the infimum in (4) equals $\mathcal{C}$ on this finest chain. Each summand in (3) is non-negative under co-monotonicity, so $\mathcal{C} = 0$ iff each summand vanishes, iff each factor is zero ν-a.e. $\square$

The finite-type result is a testable condition for discrete mechanism design. If there exists a good k for which both the valuation and the allocation change between two consecutive types, the mechanism designer has a free parameter in the payment. Different incentive-compatible payment schedules can coexist with the same allocation. On an interval, the no common discontinuities condition can be satisfied by continuity of v_k even though both v_k and q_k change at every point. On a finite set, there is no limiting argument available and the condition must hold step by step.

When uniqueness fails, the degree of non-uniqueness can be quantified. On a finite type space $\Sigma = \{s_1 < \cdots < s_n\}$ with finitely many goods, the payment p is a node potential satisfying $p(s_i) - p(s_{i-1}) \in [\ell_i^-, \ell_i^+]$ for each consecutive pair. Here ℓ_i^- and ℓ_i^+ are the lower and upper IC bounds. When v_k and q_k both change between s_{i-1} and s_i for some k, the interval $[\ell_i^-, \ell_i^+]$ has positive width, contributing one degree of freedom. If m consecutive pairs violate the uniqueness condition, the set of incentive-compatible payments for the given allocation is an m-dimensional polytope. On an interval type space, each common discontinuity of v_k and q_k contributes one degree of freedom. If there are J such points (across all goods), the payment has J free parameters.[4]

2.2 Applications

Four corollaries apply the theorem to standard settings. The first recovers classical revenue equivalence. The second and third cover multi-unit auctions and nonlinear pricing. The fourth establishes payoff equivalence in common-value settings, the basis for the revenue equivalence results of Section 3. All four specialize to $\Sigma = [\underline{s}, \bar{s}]$ with finitely many goods.

Corollary 6 (Classical Payment Identity)**.** *With a single good and private values*

[4] In the example with $v(s) = q(s) = \mathbf{1}(s \geq 1/2)$, there is one common discontinuity and the payment has one free parameter, $p(s) = 0$ for $s < 1/2$ and $p(s) = \alpha$ for $s \geq 1/2$, where $\alpha \in [0, 1]$.

$v(s) = s$, (5) gives $p(s) = p(\underline{s}) + \int_{\underline{s}}^{s} \sigma \, dq(\sigma)$ for any increasing allocation q. This is the payment identity of Myerson [1981] and Riley and Samuelson [1981] in auctions, and of Mussa and Rosen [1978] in nonlinear pricing. If two mechanisms produce the same allocation q, they produce the same payment. This is classical revenue equivalence.

Corollary 7 (Multi-Unit Revenue Equivalence)**.** *A bidder demands up to K homogeneous units with decreasing marginal values $v_1(s) \geq \cdots \geq v_K(s)$, each increasing in type s. Let $q_k(s)$ denote the probability of receiving at least k units.*[5] *If each q_k is increasing and has no common discontinuities with v_k, the expected payment is $p(s) = p(\underline{s}) + \sum_{k=1}^{K} \int_{\underline{s}}^{s} v_k \, dq_k$. Any two multi-unit auction formats with the same allocation probabilities $(q_1, \ldots, q_K)$ produce the same expected payment, even with discontinuous marginal values and allocation jumps. Standard multi-unit revenue equivalence arguments, as used in Ausubel [2004], require smooth marginal values. This corollary requires no differentiability.*

Corollary 8 (Nonlinear Pricing)**.** *A monopolist sells a product with K quality dimensions to a buyer with type s and utility $\sum_{k=1}^{K} v_k(s) q_k - T$, where T is the tariff. If each v_k is increasing and the optimal quality schedule has each $q_k(s)$ increasing in s with no common discontinuities with v_k, the tariff schedule is uniquely determined: $T(\mathbf{q}(s)) = T(\mathbf{q}(\underline{s})) + \sum_{k=1}^{K} \int_{\underline{s}}^{s} v_k \, dq_k$. No matter how the monopolist designs the menu, the payment schedule is pinned down by the quality schedule. With $K = 1$, this recovers the tariff identity of Mussa and Rosen [1978]. It complements Rochet and Choné [1998], who study multi-dimensional types with multi-dimensional quality under smoothness. Our result handles one-dimensional types with multi-dimensional*

[5] If the bidder receives exactly j units with probability $\pi_j(s)$, the expected payoff is $\sum_{j=1}^{K} \pi_j(s) \sum_{k=1}^{j} v_k(s) - p(s) = \sum_{k=1}^{K} v_k(s) q_k(s) - p(s)$, where $q_k(s) = \sum_{j \geq k} \pi_j(s)$ is the probability of at least k units.

quality and requires no differentiability.[6]

Corollary 9 (Payoff Equivalence)**.** *With a single good, suppose the object has common value V and the agent's valuation is $v(s) = \mathbb{E}[V \mid S = s]$. If v and q are both increasing with no common discontinuities, the agent's interim payoff is uniquely determined by the allocation*

$$\pi(s) = \pi(\underline{s}) + \int_{\underline{s}}^{s} q \, dv.$$

This holds in any mechanism, for any number of agents, and under any information structure of opponents.

Proof. By Corollary 4, $p(s) = p(\underline{s}) + \int_{\underline{s}}^{s} v \, dq$. Stieltjes integration by parts gives $\int_{\underline{s}}^{s} v \, dq + \int_{\underline{s}}^{s} q \, dv = v(s)q(s) - v(\underline{s})q(\underline{s})$, so $\pi(s) = v(s)q(s) - p(s) = \pi(\underline{s}) + \int_{\underline{s}}^{s} q \, dv$. $\square$

Payoff equivalence says that in any common-value environment, the allocation pins down not just the payment but the agent's surplus function. Prior revenue equivalence results rely on the envelope theorem and require differentiability or absolute continuity of the allocation [Milgrom and Segal, 2002]. Our result requires no regularity beyond the absence of common discontinuities, and applies beyond auctions to any mechanism

[6] The separable multi-good payoff encompasses several other classical models. In the regulation model of Baron and Myerson [1982], a firm of cost type θ produces output $q(\theta)$ and receives transfer $t(\theta)$. The firm's payoff $t - c(\theta)q$ is our framework with $v(\theta) = c(\theta)$ and goods = output. In scoring auctions, a bidder with type s offers a quality vector $\mathbf{q}$ and a price p. The buyer evaluates $\sum_k w_k q_k - p$, and incentive compatibility of the bidder's problem gives our separable form. In dynamic mechanisms with persistent types ($s_t = s$ for all t), the agent's discounted payoff $\sum_{t=1}^{T} \delta^{t-1} v_t(s) q_t(s) - P(s)$ is our framework with goods indexed by time periods. Our theorem pins down the total payment $P(s)$ given the allocation path $\{q_t(s)\}_{t=1}^{T}$, recovering the persistent-type case of Pavan et al. [2014] without their smoothness assumptions. In bilateral trade, a buyer with private value v_B faces a trading probability $q(v_B)$ and a transfer $p_B(v_B)$. Payment uniqueness pins down p_B, and together with the seller's payment identity, budget balance yields a single constraint that drives the impossibility result of Myerson and Satterthwaite [1983]. In matching markets, a worker of type s is assigned to a firm with probability $q(s)$ and receives wage $p(s)$. If the worker's value from a match is $v(s)q(s) - p(s)$, payment uniqueness pins down the wage schedule.

with separable payoffs.[7] The one-insider model of Section 3 is where payoff equivalence has its sharpest consequences. There, allocation invariance ($q = G$ in every BDS equilibrium) converts payoff equivalence into revenue equivalence.

3 Revenue Equivalence in Common-Value Auctions

The one-insider common-value model provides a new application of payment uniqueness where the theorem resolves an open question. The model, due to Wilson [1967, 1969] and Engelbrecht-Wiggans et al. [1983], is the simplest common-value auction with asymmetric information and has a long history of theoretical and empirical study. The open question of revenue equivalence across formats reduces, via payment uniqueness, to one of allocation invariance. Throughout this section, $\mathcal{K}$ is a singleton. The general valuation v specializes to $\mu(s) = \mathbb{E}[V \mid S = s]$, the allocation q becomes the insider's winning probability $H(s)$, and $\Sigma = [\underline{s}, \bar{s}]$.

3.1 The One-Insider Model

A single indivisible object of common value $V \geq 0$ (with $\mathbb{E}[V] < \infty$) is sold by auction to $n \geq 2$ risk-neutral bidders. Bidder 1 (the insider) observes a private signal S. Bidders $2, \ldots, n$ (uninformed) observe nothing.

Assumption 1. S takes values in $[\underline{s}, \bar{s}]$ with continuous CDF G.

Assumption 2. $\mu(s) := \mathbb{E}[V \mid S = s]$ is continuous and strictly increasing on $[\underline{s}, \bar{s}]$, with $\mu(\underline{s}) \geq 0$.

[7] In bilateral trade, a buyer with signal s about a common-value object faces a trading probability $q(s)$ and a transfer $p(s)$. If $v(s) = \mathbb{E}[V \mid S = s]$ and q are increasing with no common discontinuities, the buyer's surplus is determined by the trading rule regardless of the mechanism. In labor market screening, where a firm sets a hiring probability $q(s)$ and a wage $p(s)$ based on a signal s about worker productivity, the wage schedule is determined by the hiring rule.

Strict monotonicity of μ, which was not needed for payment uniqueness, is used here to obtain strict increasing differences in the insider's payoff.

Definition 10 (Standard Auction Format)**.** A sealed-bid mechanism with payment functions (ρ_w, ρ_ℓ). The highest bidder wins (ties broken uniformly). The winner pays $\rho_w(b_w, \mathbf{b}_{-w})$, each loser pays $\rho_\ell(b_\ell, \mathbf{b}_{-\ell})$, and $\rho_\ell(0, \cdot) = 0$.

3.2 BDS Equilibrium and Allocation Invariance

Definition 11 (BDS Equilibrium)**.** An equilibrium is *bid-distribution symmetric* (BDS) if the insider employs a pure strategy β such that the CDF of the maximum uninformed bid equals the CDF of the insider's bid.

Under BDS with increasing β, the insider who observes s and deviates to $\beta(z)$ wins with probability $G(z)$. The expected payoff is $U(z, s) = \mu(s)G(z) - p^{\mathcal{A}}(z)$, where $p^{\mathcal{A}}(z)$ is the format-specific expected payment when bidding $\beta(z)$. The payoff has strict increasing differences in (z, s)

$$U(z', s') - U(z, s') - U(z', s) + U(z, s) = [\mu(s') - \mu(s)][G(z') - G(z)] > 0.$$

In any BDS equilibrium, therefore, the insider's strategy is increasing and the winning probability is $H(s) = G(s)$, regardless of the format. This is allocation invariance.

3.3 Revenue Equivalence

Theorem 12 (Revenue Equivalence)**.** *In the one-insider model, every BDS equilibrium satisfies*

(i) $p(s) = \int_{\underline{s}}^{s} \mu \, dG$*, independently of the format.*

(ii) $\pi(s) = \int_{\underline{s}}^{s} G\,d\mu$.

(iii) $\Pi_1 = \int_{\underline{s}}^{\bar{s}} G(1-G)\,d\mu$.

(iv) $R = \mathbb{E}[V] - \Pi_1$*, independently of the format.*

Proof. (i) Corollary 4 with $H = G$ and $p(\underline{s}) = 0$ (the lowest type bids zero, wins with probability zero, and $\rho_\ell(0,\cdot) = 0$).

(ii) By Stieltjes integration by parts (with $G(\underline{s}) = 0$) $\int_{\underline{s}}^{s} \mu\,dG + \int_{\underline{s}}^{s} G\,d\mu = \mu(s)G(s)$, so $\pi(s) = \mu(s)G(s) - p(s) = \int_{\underline{s}}^{s} G\,d\mu$.

(iii) $\Pi_1 = \int \pi\,dG = \int [\int_{\underline{s}}^{s} G\,d\mu]\,dG(s)$. By Fubini $\Pi_1 = \int G(1-G)\,d\mu$.

(iv) Total expected surplus is $\mathbb{E}[V]$ (common value). In any BDS equilibrium, uninformed bidders earn zero. By anonymity, an active uninformed bidder at bid $\beta(s)$ faces the same strategic situation as the insider at bid $\beta(s)$, so the expected payment is $p(s)$. The expected value of winning is $\mathbb{E}[V \cdot \mathbf{1}(S \leq s)] = p(s)$ by the tower property.[8] Hence $R = \mathbb{E}[V] - \Pi_1$. □

Revenue equivalence thus combines two independent facts. The first is payment uniqueness (Theorem 1), which holds for any continuous increasing pair (μ, H). The second is allocation invariance ($H = G$ in every BDS equilibrium, regardless of format). Neither alone suffices. Without allocation invariance, different formats could produce different allocations, and payment uniqueness would give a different payment for each. Without payment uniqueness, the common allocation $H = G$ might be compatible with multiple incentive-compatible payment schedules, and different formats could select different ones.

[8] In first-price, zero profit extends to any factorization of the uninformed bid CDF. In second-price and all-pay, every BDS equilibrium has a single active uninformed bidder. With multiple active uninformed, each earns strictly negative surplus due to the winner's curse among uninformed. See Appendix A.

As an illustration, consider $S \sim \text{Uniform}[0,1]$ with $\mu(s) = s^2$. The insider's ex ante profit is $\Pi_1 = \int_0^1 \sigma(1-\sigma) \cdot 2\sigma \, d\sigma = 1/6$, and the seller's revenue is $R = \mathbb{E}[S^2] - 1/6 = 1/6$.

Theorem 13 (Existence)**.** *Under Assumptions 1–2, for any standard auction format and any $n \geq 2$, a BDS equilibrium exists with increasing insider bid function β.*

The proof (Appendix A) derives the bid function from payment uniqueness, verifies global optimality using monotonicity of μ, and constructs supporting strategies for the uninformed bidders.[9]

Corollary 14 (Orthogonality)**.** *In the one-insider model, the seller's BDS revenue $R = \mathbb{E}[V] - \int G(1-G)\, d\mu$ depends on the signal structure (G, μ) but not on the standard auction format. The format choice and the disclosure policy are independent.*

If the seller discloses S publicly, competitive bidding drives revenue to $\mathbb{E}[V]$. The value of disclosure is therefore $\Pi_1 = \int G(1-G)\, d\mu$ in every format, extending Milgrom and Weber [1982b] from first-price to all standard formats. The government can choose the auction format for administrative reasons (simplicity, legal precedent, susceptibility to collusion) and independently optimize its disclosure policy for revenue.[10]

[9] The bid functions are: first-price, $\beta_{FP}(s) = G(s)^{-1} \int_{\underline{s}}^{s} \mu \, dG = \mathbb{E}[\mu(S) \mid S \leq s]$; second-price, $\beta_{SP}(s) = \mu(s)$; all-pay, $\beta_{AP}(s) = \int_{\underline{s}}^{s} \mu \, dG$. All three yield expected payment $p(s) = \int_{\underline{s}}^{s} \mu \, dG$. The first-price function appears in EMW. The all-pay function in Grosskopf et al. [2010]. In second-price, $\beta_{SP} = \mu$ is weakly dominant. With $n = 2$, every uninformed strategy is a best response, generating revenues anywhere in $[0, \mathbb{E}[V]]$. In first-price, the BDS equilibrium is unique [Engelbrecht-Wiggans et al., 1983].

[10] The orthogonality is strongest when the format has a unique equilibrium (as in first-price, where BDS is the unique equilibrium by Engelbrecht-Wiggans et al., 1983, Theorem 1). In formats with multiple equilibria (second-price), the format choice may affect which equilibria are available even if BDS revenue is the same.

3.4 Multiple Insiders and Revenue Decomposition

Corollary 4 applies to each informed bidder individually. If insider i has conditional value $\mu_i(s_i) = \mathbb{E}[V \mid S_i = s_i]$ continuous and strictly increasing, and equilibrium allocation H_i increasing, then $p_i(s_i) = p_i(\underline{s}_i) + \int \mu_i \, dH_i$. With multiple insiders, however, there is no analog of BDS that forces $H_i = G_i$ regardless of format. Allocation invariance generically fails.

Proposition 15 (Revenue Decomposition)**.** *Consider n informed bidders (no uninformed) in two mechanisms $\mathcal{A}$ and $\mathcal{B}$ with monotone equilibria and equal boundary payments. The revenue difference is*

$$R^{\mathcal{A}} - R^{\mathcal{B}} = \sum_{i=1}^{n} \int \mu_i(\sigma)[1 - G_i(\sigma)] \, d(H_i^{\mathcal{A}} - H_i^{\mathcal{B}})(\sigma). \tag{6}$$

When $H_i^{\mathcal{A}} = H_i^{\mathcal{B}}$ for all i, each term vanishes and revenue equivalence holds.

Proof. Corollary 4 gives $p_i(s_i) = p_i(\underline{s}_i) + \int_{\underline{s}_i}^{s_i} \mu_i \, dH_i$. Taking expectations and applying Fubini: $\mathbb{E}[p_i] = p_i(\underline{s}_i) + \int \mu_i(1 - G_i) \, dH_i$. Sum and take differences. □

The decomposition of revenue equivalence into payment uniqueness and allocation invariance unifies several classical results. In symmetric IPV [Myerson, 1981], $v(s) = s$ and symmetry gives allocation invariance, so revenue equivalence holds. In the one-insider model, $v = \mu$ and BDS gives $H = G$, so revenue equivalence holds. In asymmetric multi-insider settings [Maskin and Riley, 2000], payment uniqueness holds but allocation invariance fails. The signed measures $d(H_i^{\mathcal{A}} - H_i^{\mathcal{B}})$ in (6) are nonzero, and revenue equivalence breaks down. In each case, the same two-condition framework applies, and what changes is whether allocation invariance holds.

4 Concluding Remarks

Separability plays two roles in the proof of payment uniqueness. The first, monotone path optimality, requires only increasing differences. For any payoff $W(s,q) - p$ with W having increasing differences in (s,q), the type graph has arc lengths $\ell_{st} = W(t,q(t)) - W(t,q(s))$. Each of the inequalities (a)–(d) of Lemma 2 then reduces to the sign of a mixed difference of W, which is non-negative by increasing differences. Monotone paths remain optimal.

The second role, anti-symmetry, is more demanding. The round-trip cost along any partition $\underline{s} = \sigma_0 < \cdots < \sigma_m = \bar{s}$ equals the sum of mixed differences

$$\sum_{i=1}^{m} \left[W(\sigma_i, q_i) + W(\sigma_{i-1}, q_{i-1}) - W(\sigma_i, q_{i-1}) - W(\sigma_{i-1}, q_i)\right],$$

each term non-negative under increasing differences. Anti-symmetry requires this sum to converge to zero as the partition refines. In the separable case, each term factors as $[v(\sigma_i) - v(\sigma_{i-1})][q_i - q_{i-1}]$, and the sum converges to zero if and only if v and q have no common discontinuities, recovering Corollary 4.

In general, convergence to zero can fail even for continuous payoffs with increasing differences. Let $W(s,q) = \min(s,q)$ and $q(s) = s$ on $[0,1]$. The mixed difference on each cell equals $\sigma_i - \sigma_{i-1}$, so the round-trip cost sums to 1 regardless of the partition. Anti-symmetry fails, and the payment is not unique. Smoothness can substitute for separability. If W is C^1 with increasing differences, each mixed difference satisfies $|\Delta_i| \leq (\sigma_i - \sigma_{i-1}) \cdot \sup |W_s(\cdot, q_i) - W_s(\cdot, q_{i-1})|$, and the sum vanishes by uniform continuity of W_s, restoring uniqueness. Separability is therefore the weakest condition that guarantees payment uniqueness without differentiability. Under separability, the no-common-discontinuities condition suffices. Without it, even continuity of W does not.

Under risk aversion, the difficulty is more fundamental. The agent's expected utility takes the form $q \cdot u(v(s) - p_w) + (1-q) \cdot u(-p_\ell)$, where the payment appears inside the utility function rather than as an additive term. The payoff no longer decomposes as $W(s,q) - p$. The type-graph framework does not apply. Increasing differences suffice for monotone paths, separability or smoothness for anti-symmetry, additive payment for the node-potential structure itself. It is the payoff structure, not the information structure, that determines whether revenue equivalence holds.[11]

The multi-good formulation (Theorem 1) requires the payoff to be additive across goods, $\int_{\mathcal{K}} v_k(s) q_k \, d\nu(k) - p$. This excludes complementarities and substitutabilities between goods, where the marginal value of good k depends on the allocation of other goods. A payoff with cross-good interactions does not decompose into a sum of independent one-dimensional terms, the arc length ℓ_{st} cannot be written as $\int_{\mathcal{K}} [\cdot][q_k(t) - q_k(s)] \, d\nu(k)$, and the sign conditions of Lemma 2 fail. This is the multi-good analog of the non-separable payoff $W(s,q) - p$ discussed above. The additive structure across goods is essential, just as separability in the signal is essential within each good.

The "goods" in the theorem need not be physically separate objects. In the multi-unit formulation (Corollary 7), the goods are "at least k units," which incorporate declining marginal values from substitutability between identical units. The additive restriction binds only when goods have cross-good complementarities or substitutabilities that cannot be repackaged into an additive form.

The multi-unit corollary establishes payment uniqueness for multi-unit auctions,

[11] Payment uniqueness is specific to one-dimensional signals. With $S \in \mathbb{R}^d$ for $d \geq 2$, anti-symmetry requires path-independence, which generally fails [Krishna and Maenner, 2001]. The result extends immediately to signals with mass points (the Stieltjes integral accommodates atoms in q), though the BDS construction in Section 3 relies on a continuous CDF G.

but revenue equivalence requires a second ingredient, allocation invariance across formats. In the single-unit one-insider model, the BDS structure forces $H = G$ because the highest-bid-wins rule pins down the insider's winning probability at each bid level. In multi-unit auctions, the mapping from demand schedules to allocation probabilities $(q_1, \ldots, q_K)$ depends on the auction format. In a discriminatory auction, the insider optimizes each unit's bid independently, while in a uniform-price auction, strategic demand reduction alters the equilibrium quantities. Further, in a Vickrey auction, truth-telling is dominant. The resulting allocation vectors generically differ across formats, even under the one-insider information structure. Payment uniqueness provides one half of multi-unit revenue equivalence. Whether allocation invariance provides the other half in specific multi-unit one-insider environments is an open question.

The common-value model analyzed in Section 3 is a special case of the interdependent-values framework studied by Milgrom and Weber [1982a], Dasgupta and Maskin [2000], and Jehiel and Moldovanu [2001]. Payment uniqueness applies whenever one bidder's interim payoff takes the separable form, which holds in any environment where $\mathbb{E}[V \mid S_i = s_i]$ is increasing. It does not extend to settings where bidder i's value depends directly on opponents' signals in a non-separable way, as in the general model of Jehiel and Moldovanu [2001]. The orthogonality result (Corollary 14) connects to the information design literature of Bergemann et al. [2017]. The seller's problem of choosing (G, μ) to maximize $\mathbb{E}[V] - \int G(1-G)\, d\mu$ is independent of the auction format.

Theorem 1 completes a chain in the theory of incentive compatibility. Archer and Tardos [2001] characterize when an allocation is *implementable*, showing that in single-parameter domains, an allocation admits at least one incentive-compatible payment if and only if it is monotone. Saks and Yu [2005] extend this to convex type domains. HMUV [2009] provide a criterion for when the payment is unique, namely anti-symmetry of shortest-path distances. We verify HMUV's criterion for separable payoffs, obtaining a sharp condition (the round-trip cost). Monotonicity determines whether a payment exists. The round-trip condition determines whether it is unique. When it is unique, the shortest-path distance gives its value.

A Existence of BDS Equilibria

Proof of Theorem 13. Fix a standard auction format (ρ_w, ρ_ℓ) and $n \geq 2$.

Step 1: Candidate bid function. Suppose a BDS equilibrium exists with increasing β. Then $H(s) = G(s)$, and Corollary 4 gives $p(s) = \int_{\underline{s}}^{s} \mu \, dG$. Under Assumptions 1–2, p is continuous and strictly increasing (since μ is strictly increasing and G has full support). The bid function β is the unique function such that the expected payment at bid $\beta(s)$, given winning probability $G(s)$ and opponent bid CDF $G \circ \beta^{-1}$, equals $p(s)$. Since p is continuous and strictly increasing, β inherits both properties.

Step 2: Global optimality for the insider. The insider who reports z earns $U(z, s) = \mu(s)G(z) - \int_{\underline{s}}^{z} \mu \, dG$. For $z < s$: $U(s, s) - U(z, s) = \mu(s)[G(s) - G(z)] - \int_{z}^{s} \mu \, dG \geq 0$, since $\mu(\sigma) \leq \mu(s)$ on $[z, s]$. For $z > s$: $U(s, s) - U(z, s) = -\mu(s)[G(z) - G(s)] + \int_{s}^{z} \mu \, dG \geq 0$, since $\mu(\sigma) \geq \mu(s)$ on $[s, z]$. Truth-telling is globally optimal.

Step 3: Uninformed bidders. Let bidder 2 draw $\tilde{S} \sim G$ and bid $\beta(\tilde{S})$. Assume bidders $3, \ldots, n$ bid zero.

Zero profit for the active uninformed bidder. In any standard format, the payment rule is anonymous. That is, it depends on the bids submitted, not on bidder identities. Bidder 2 at bid $\beta(s)$ faces the same strategic situation as the insider at bid $\beta(s)$, with one opponent whose bid has CDF $G \circ \beta^{-1}$ and $n-2$ opponents bidding zero. By anonymity, bidder 2's expected payment at $\beta(s)$ equals $p(s)$. The expected value of winning at $\beta(s)$ is $\mathbb{E}[V \cdot \mathbf{1}(S \leq s)] = \int_{\underline{s}}^{s} \mu \, dG = p(s)$, by the tower property. Zero surplus at every bid in the support. This argument is format-independent.

Inactive bidders. Each inactive bidder bids zero, pays zero (since $\rho_\ell(0, \cdot) = 0$), and wins with probability zero, earning zero surplus. An inactive bidder who deviates to $\beta(z)$ must beat both the insider and bidder 2, winning with probability $G(z)^2 < G(z)$. The expected value conditional on winning is unchanged ($\mathbb{E}[V \mid S \leq z, \tilde{S} \leq z] = \mathbb{E}[V \mid S \leq z]$, since $\tilde{S}$ carries no information about V). The payment weakly rises. In formats where the winner's payment depends only on the winner's own bid, the payment is the same but the winning probability is strictly lower. In formats where the payment depends on losing bids, the additional opponent's bid can only raise the price. Since bidder 2 earns zero surplus at $\beta(z)$, the inactive bidder earns non-positive surplus at every deviation. $\square$

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